

# Configurational Entropy in Brane-world Models: A New Approach to Stability

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In this work we investigate the entropic information on thick brane-worlds scenarios and its consequences. The brane-world entropic information is studied for the sine-Gordon model and hence the brane-world entropic information measure is shown an accurate way for providing the most suitable values for the bulk AdS curvature. Besides, the brane-world configurational entropy is employed to demonstrate a high organisational degree in the structure of the system configuration, for large values of a parameter of the sine-Gordon model but the one related to the AdS curvature. The Gleiser and Stamatiopoulos procedure is finally applied in order to achieve a precise correlation between the energy of the system and the brane-world configurational entropy.

PACS numbers: 11.25.-w, 11.27.+d, 89.70.Cf

Keywords: entropy, sine-Gordon model, brane-world models, gravity, topological defects

## I. INTRODUCTION

The 4D Universe can be regarded as a brane embedded in a higher-dimensional bulk, in which extra dimensions can be large [1, 2] and either compact or non-compact [3–7]. Brane-world models have been providing new tools to understand various questions, as for instance the solution of the gauge hierarchy problem [1, 5, 6].

Recently, an widening interest has been focused on the study of thick brane scenarios based on gravity coupled to one or more scalars in higher dimensional spacetimes [8–26]. Moreover, domain walls have been used in high-energy physics to generate thick branes in models wherein scalar fields can couple with gravity in warped spacetime. Besides, thick branes are well-known to avoid some inconsistencies inherent to thin brane-worlds, as for instance the proton decay [27]. For some prominent reviews on domain walls and thick branes see, e. g., Refs. [28, 29]. In particular, a model described by a single scalar field with internal structure was proposed in Refs. [17, 30]. Analytical solutions of the Einstein equations were achieved with a sine-Gordon potential [31], where a kink, as the scalar field configuration in the system, provides the thick brane-world as a domain wall in the bulk. Similarly, this type of configuration has been also further explored [32, 33]. In addition, an analytic solution of the sine-Gordon domain wall in a 4D global supersymmetric model was obtained [34], and the stability of a more general setup was studied likewise [35]. The localization of fermions on the brane has been accomplished in the presence of kink-fermion couplings in the background of the sine-Gordon kink [25].

Our prominent aim here is to study the entropic infor-

mation on thick brane-worlds models, by means of the brane-world configurational entropy, and to further explore its subsequent ramifications. In order to accomplish it, the sine-Gordon kink shall be used, playing a prominent role on the thick brane scenario.

This paper is organized as follows: in the next section the thick brane generated by a single scalar field coupled to gravity shall be revisited, with particular attention to the sine-Gordon model. In Sec. III the brane-world configurational entropy is going to be introduced, and we shall calculate the entropic information for the sine-Gordon model, using the Fourier transform of the thick brane (warped) energy density. The entropic information measure shall be shown a successful manner for constraining the most suitable values for the AdS curvature. In addition, the configurational entropy is employed to evince a high organisational degree in the configuration of the system, for large values of a parameter of the sine-Gordon model. Furthermore, the Gleiser and Stamatiopoulos procedure is going to be applied to obtain a correlation between the brane-world configurational entropy and the energy of the system. In the last section we explicit the concluding remarks.

## II. A BRIEF OVERVIEW OF GRAVITY COUPLED TO A SCALAR FIELD

In this section the work presented by Gremm is briefly revisited [9], where 4-dimensional gravity arises on a thick domain wall in AdS space. We start with the action in 5-dimensional gravity coupled to one real scalar field, which is given by

$$\mathcal{S} = \int d^5x \sqrt{|g|} \left[ -\frac{R}{4} + \frac{1}{2} g^{ab} \nabla_a \phi \nabla_b \phi - V(\phi) \right], \quad (1)$$

where  $4\pi G = 1$ , with the field and the space-time variables being dimensionless,  $R$  denotes the scalar curva-

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ture, and the scalar field  $\phi$  depends only on the extra dimension. Furthermore,  $V(\phi)$  is the potential describing the model,  $g = \det(g_{ab})$ , and the metric is represented by

$$ds^2 = g_{ab}dx^a dx^b = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2, \quad (2)$$

for  $a, b = 0, 1, 2, 3, 5$ , where  $r$  is the extra dimension,  $\eta_{\mu\nu}$  denotes the usual Minkowski metric components, and  $e^{2A(r)}$  is the warp factor.

By denoting  $B'(r) = dB(r)/dr$ , for any quantity  $B$  depending just upon the variable  $r$ , and using the Einstein equations  $\mathcal{G}_{ab} = \mathcal{T}_{ab}$  and the Euler-Lagrange equation  $\nabla\phi_a \nabla\phi^a + V'(\phi) = 0$  as well, the following equations of motion read:

$$A''(r) + \frac{2}{3}\phi'^2(r) = 0, \quad (3)$$

$$A'^2(r) - \frac{1}{6}\phi'^2(r) + \frac{1}{3}V(\phi) = 0, \quad (4)$$

$$\phi''(r) + 4\phi'(r)A'(r) - V'(\phi) = 0. \quad (5)$$

Moreover, the above equations are equivalently written as

$$\phi'(r) = \frac{1}{2} \frac{dW(\phi)}{d\phi}, \quad A'(r) = -\frac{1}{3}W(\phi) \quad (6)$$

whenever the potential  $V(\phi)$  is provided by the superpotential  $W(\phi)$  as [8].

$$V(\phi) = \frac{1}{8} \left( \frac{dW(\phi)}{d\phi} \right)^2 - \frac{1}{3}W^2(\phi). \quad (7)$$

Thus, it is straightforward to verify that the first-order equations

$$A'(r) = -\frac{1}{3}W(\phi), \quad (8)$$

$$\phi'(r) = \frac{1}{2} \frac{dW(\phi)}{d\phi}, \quad (9)$$

also solve Eqs. (3), (4) and (5). In order to find analytical solutions, the sine-Gordon model is employed [36, 37], being defined by the following superpotential:

$$W(\phi) = 3\alpha\beta \sin \left( \sqrt{\frac{2}{3\alpha}}\phi \right). \quad (10)$$

By using the above equation in Eq. (7), the potential yields

$$V(\phi) = \frac{3\alpha\beta^2}{8} \left[ (1 - 4\alpha) - (1 + 4\alpha) \cos \left( \sqrt{\frac{8}{3\alpha}}\phi \right) \right]. \quad (11)$$

Now the solutions of Eqs. (8) and (9) are straightforwardly verified to be given by

$$A(r) = -\alpha \ln [2 \cosh(\beta r)], \quad (12)$$

$$\phi(r) = \sqrt{6\alpha} \arctan \left[ \tanh \left( \frac{\beta r}{2} \right) \right]. \quad (13)$$

The field  $\phi(r)$  and the warp factor  $e^{2A(r)}$  are shown, respectively, in Figs. 1 and 2. The field configuration in Fig. 1 is evinced to be the so-called kink. Moreover,  $e^{2A(r)}$  is centred on  $r = 0$ . It is important to remark that in the solutions (12) and (13) the AdS curvature is related by the product  $\alpha\beta$ , whereas the thickness of the domain wall is given by the parameter  $\beta$ . In addition, the brane core is localized at  $r = 0$ , which is consonant to the point of maximum variation of the scalar field, what composes the well-known domain wall. The thick brane presents a thickness  $\Delta$  where deviations from the 4D Newton's law eventuate in these scales. Current precision torsion-balance experiments require that the extra dimension must satisfy the constraint  $\Delta \lesssim 44 \mu\text{m}$  [38], whereas theoretical reasons thus imply that  $\Delta \gtrsim \ell_{(5)} \approx 2.0 \times 10^{-19} \text{m}$ . Indeed, since in the 5D scenario with  $M_{(5)} \simeq M_{\text{ew}} \simeq 1 \text{TeV}$ , to be considered, the electroweak scale corresponds to the length  $\ell_{(5)} \simeq 2.0 \times 10^{-19} \text{m}$ . Besides, it was shown in Ref. [27] that the above experimental and theoretical arguments impose physical constraints for the parameter space  $\alpha\beta$ :

$$1.0 \times 10^{-19} \beta \lesssim \text{arccosh} \left( \frac{10^{13/\alpha}}{2} \right) \lesssim 2.2 \times 10^{-5} \beta. \quad (14)$$

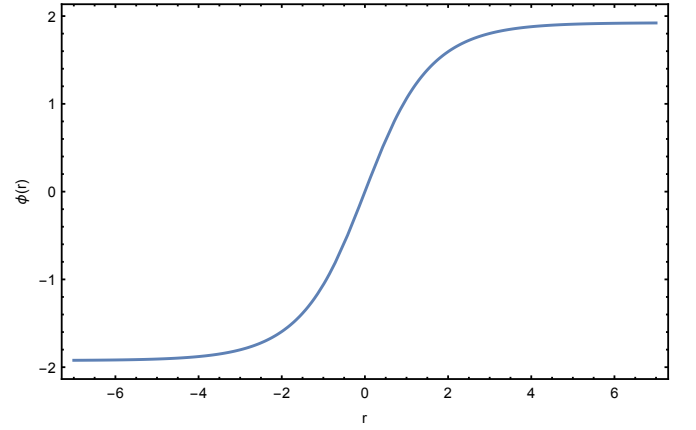


FIG. 1: Kink-like solution for  $\alpha = \beta = 1$ .

In the next section we shall postulate the configurational entropy in brane-world scenarios. As an example, the sine-Gordon model described here shall be explored.

### III. BRANE-WORLD CONFIGURATIONAL ENTROPY (BCE)

As argued in the Introduction, Gleiser and Stamatopoulos (GS) [39] have recently proposed a detailed picture of the so-called Configurational Entropy (CE) for the structure of localized solutions in classical field theories. In this section, analogously to that work, we formulate a configurational entropy measure in the functional

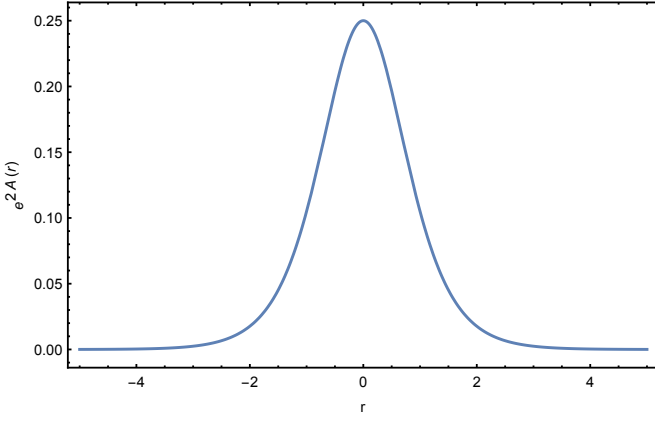


FIG. 2: Warp factor with  $\alpha = \beta = 1$ .

space, from the field configurations where brane-world scenarios can be studied. Firstly, the framework shall be formally introduced and thereafter its consequences are going to be explored. Hence, we discuss a prominent feature of this theory.

To start, let us write the following Fourier transform

$$\mathcal{F}[\omega] = -\frac{1}{\sqrt{2\pi}} \int dr e^{2A(r)+i\omega r} \mathcal{L}, \quad (15)$$

where  $\mathcal{L}$  is the standard Lagrangian density and  $e^{2A(r)}$  denotes the warp factor, as usual. For the sake of simplicity, define  $\varepsilon(r) := -\mathcal{L}e^{2A(r)}$ , named the warp density (WD). Thus, using the Plancherel theorem it follows that

$$\int d\omega |\mathcal{F}[\omega]|^2 = \int dr |\varepsilon(r)|^2. \quad (16)$$

Now the modal fraction is defined by the following expression [39–43]:

$$f(\omega) = \frac{|\mathcal{F}[\omega]|^2}{\int d\omega |\mathcal{F}[\omega]|^2}, \quad (17)$$

The modal fraction  $f(\omega)$  measures the relative weight of each mode  $\omega$ . Analogously to the Shannon's information theory, the CE can be described by the expression

$$S_c[f] = - \int d\omega \tilde{f}(\omega) \ln[\tilde{f}(\omega)], \quad (18)$$

where  $\tilde{f}(\omega) := f(\omega)/f_{\max}(\omega)$  is defined as the normalized modal fraction, whereas the term  $f_{\max}(\omega)$  denotes the maximum fraction. Thus, Eq. (15) can be used to generate the modal fraction, in order to obtain the entropic profile of thick brane solutions. It is important to remark that Eq. (15) differs from that one given by GS. In this framework we are including the warp effect in the function  $\mathcal{F}[\omega]$ , and consequently the framework carries information about the warped geometry.

Here, as a straightforward example, we shall calculate the entropic information for the sine-Gordon model. By

substituting Eq. (7) and Eq. (9) as well into the WD, and after straightforward manipulations, it yields

$$\varepsilon(r) = e^{2A} \left( \frac{1}{4} W_\phi^2 - \frac{1}{3} W^2 \right). \quad (19)$$

With the sine-Gordon model provided by Eq. (10) and its respective solutions, the above WD can be written in the following form:

$$\varepsilon(r) = \frac{6\alpha\beta^2 \cosh^{-2}(\beta r)}{[1 + \tanh^2(\beta r)]^2} [\operatorname{sech}^4(\beta r) - 6\alpha \tanh^2(\beta r)]. \quad (20)$$

Now, the Fourier transform of the WD is calculated, which gives the modal fraction in Eq. (17). In fact, we must determine

$$\mathcal{F}[\omega] = \frac{1}{\sqrt{2\pi}} \int dr e^{i\omega r} \varepsilon(r), \quad (21)$$

with  $\varepsilon(r)$  given by Eq. (20). After exhaustive calculations, one finds

$$\mathcal{F}[\omega] = \frac{2^{1-2\alpha}\alpha\beta^2}{\sqrt{2\pi}} \sum_{j=1}^2 \left[ \frac{2(2+9\alpha)}{3} I_1^{(j)} - \frac{3\alpha}{4} I_2^{(j)} \right], \quad (22)$$

where  $I_1^{(j)}$  and  $I_2^{(j)}$  are functions given by

$$I_1^{(j)} = \frac{1}{2\beta} \frac{\Gamma(\lambda_j + 1)}{\Gamma(\lambda_j + 2)} {}_2G_1[\gamma_j, \lambda_j + 1; \lambda_j + 2; -1], \quad (23)$$

$$I_2^{(1)} = 4(\bar{I}_2^{(1)} + \tilde{I}_2^{(1)}), \quad I_2^{(2)} = 4(\bar{I}_2^{(2)} + \tilde{I}_2^{(2)}). \quad (24)$$

The above functions are defined as

$$\bar{I}_2^{(j)} = \frac{1}{2\beta} \frac{\Gamma(\bar{\lambda}_1 + 1)}{\Gamma(\bar{\lambda}_1 + 2)} {}_2G_1[\bar{\gamma}_1, \bar{\lambda}_1 + 1; \bar{\lambda}_1 + 2; -1], \quad (25)$$

$$\tilde{I}_2^{(j)} = \frac{1}{2\beta} \frac{\Gamma(\tilde{\lambda}_1 + 1)}{\Gamma(\tilde{\lambda}_1 + 2)} {}_2G_1[\tilde{\gamma}_1, \tilde{\lambda}_1 + 1; \tilde{\lambda}_1 + 2; -1], \quad (26)$$

where the above expressions  ${}_2G_1[\cdot, \cdot; \cdot; \cdot]$  stand for the well-known hypergeometric functions with

$$\begin{aligned} \gamma_1 &= \gamma_2 = \bar{\gamma}_m = \tilde{\gamma}_m = 2(\alpha + 1), \\ \lambda_1 &= \alpha + i\omega/2\beta, \quad \lambda_2 = \lambda_1^*, \quad \bar{\lambda}_1 = \lambda_1 + 1, \\ \bar{\lambda}_2 &= \bar{\lambda}_1^*, \quad \tilde{\lambda}_1 = \lambda_1^* - 1, \quad \tilde{\lambda}_2 = \lambda_1 - 1, \end{aligned}$$

where the  $\lambda^*$  denotes the complex conjugate of  $\lambda$ . Now, in order to lead the modal fraction to a more compact form, Eq. (22) can be rewritten as:

$$\mathcal{F}[\omega] = A_0 \sum_{j,k=1}^2 c_k I_k^{(j)}, \quad (27)$$

where the following notation is used:

$$A_0 = \frac{2^{1-2\alpha}\alpha\beta^2}{\sqrt{2\pi}}, \quad c_1 = \frac{2(2+9\alpha)}{3}, \quad c_2 = -\frac{3\alpha}{4}. \quad (28)$$

Thus, the modal fraction Eq. (15) becomes

$$f(\omega) = \frac{\sum_{j,k,m,n=1}^2 c_k c_n^* I_k^{(j)} I_n^{(m)*}}{\sum_{j,k,m,n=1}^2 \int d\omega c_k c_n^* I_k^{(j)} I_n^{(m)*}}. \quad (29)$$

In Figs. 3 and 4 the modal fraction is depicted, for different values of the parameter  $\alpha$ . Note that the maximum of the distributions are localized around the zero mode  $\omega = 0$ . Indeed, the maximum fraction is given by  $f_{\max} = f_{\max}(0)$ . By taking into account the modal

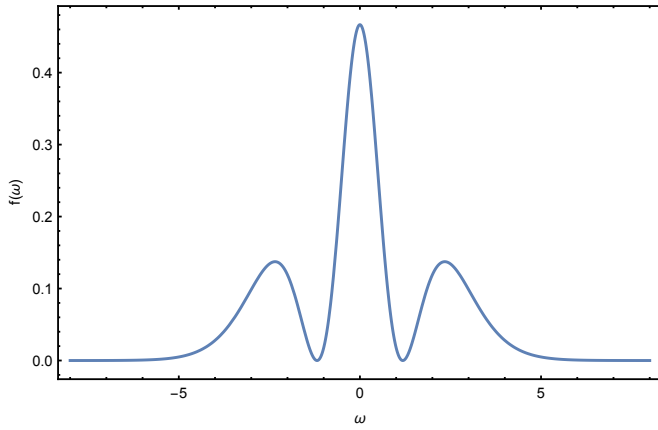


FIG. 3: Modal fractions for  $\alpha = \beta = 1$ . The maximum is at  $\omega = 0$ .

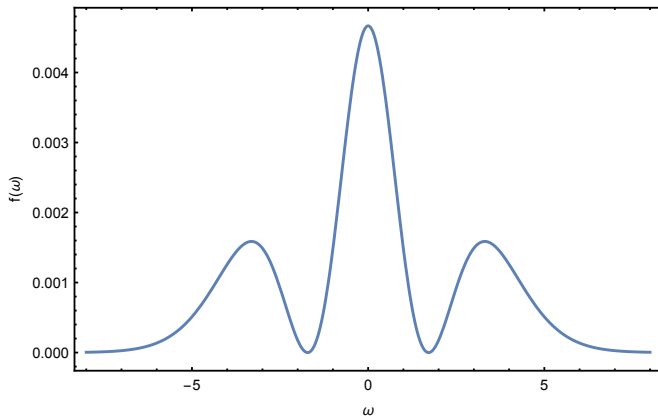


FIG. 4: Modal fractions for  $\alpha = 2$  and  $\beta = 1$ . Note that the maximum is also at  $\omega = 0$ .

fraction in Eq. (29) and its maximum contribution, Eq. (18) can be now solved in order to obtain the brane CE. In this case, due to the high complexity of integration, Eq. (18) must be integrated numerically. The results are shown in Fig. 5, where the BCE is plotted as a function of the parameter  $\alpha$ . In this case, the rescaled parameter

$\delta = \alpha\beta$  have been defined, which is responsible by the AdS curvature. Moreover, the thickness of the wall is fixed by taking  $\beta = 1, 2, 3$ . As a consequence, several remarkable results can be enumerated here. First, we have found that the entropic information measure provides an accurate way to fix the best values of the AdS curvature. In fact, we demonstrate that the best ordering is given by that ones with lower values of the domain wall thickness. Second, at large values of  $\alpha$ , the brane-world CE yields the configurational entropy  $S_c = 0$ , showing a great organisational degree in the structure of configuration of the system.

Finally, by using a recent approach presented by GS [39], we have checked that the BCE is correlated to the energy of the system. The higher [lower] the brane configurational entropy, the higher [lower] the energy of the solutions. Moreover, it is straightforward to realize from

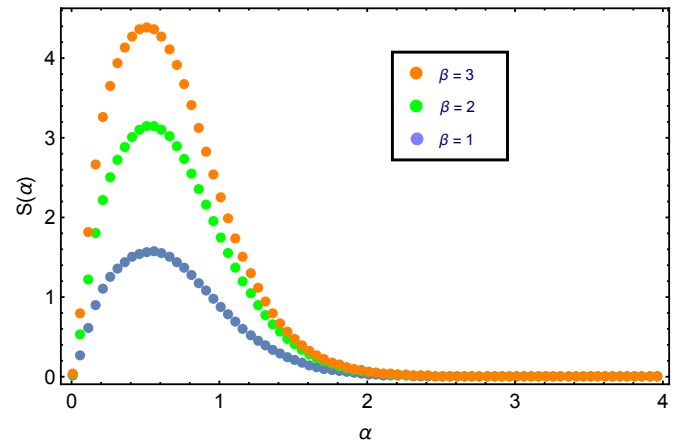


FIG. 5: The brane configurational entropy as a function of the parameter  $\alpha$ .

Fig. 5 that the higher the brane thickness  $\beta$ , the higher the respective value for the brane configurational entropy as well.

In the next section we shall provide the consequences of the model above studied and point forthcoming perspectives.

#### IV. CONCLUDING REMARKS AND OUTLOOK

The entropic information has been here studied in brane-world models, with emphasis on the sine-Gordon model, which has been chosen by its very physical content and usefulness. Indeed, the sine-Gordon model parameters provide the AdS bulk curvature and the domain wall thickness as well. Hence the brane-world entropic information for the sine-Gordon model has been achieved, providing the most suitable values for the AdS curvature. In fact, we proved that the higher the brane thickness  $\beta$ , the higher the respective value for the brane configurational entropy. The brane-world configurational entropy

is moreover exerted to evince a higher organisational degree in the structure of system configuration likewise, for large values of one of the sine-Gordon model parameters. The Gleiser and Stamatopoulos procedure was also used to acquire the correlation between the the brane-world configurational entropy and the energy of the system, withal.

Once developed the formalism of the brane-world configurational entropy and the entropic information as well, we can further apply a procedure similar to what has been studied in the previous sections to other thick brane-world models. Indeed, an entire new family of models of the sine-Gordon type, starting from the sine-Gordon model, including the double sine-Gordon, the triple one, and so on, have been obtained in [44]. Such models appear as deformations of the starting sine-Gordon model, and as they present different topological sectors, it would be important to probe them from the point of view of the brane-world configurational entropy. Since the solutions of these deformed models can be constructed explicitly from the topological defects of the sine-Gordon model

itself, we expect to study in particular the double sine-Gordon model in a brane-world scenario with a single extra dimension of infinite extent, as in this framework a stable gravity scenario has been shown to be allowable [44]. Other interesting brane-world models that are beyond the scope here, as for instance the Bloch branes [18], the cyclically deformed topological defects that generate domain walls [47], and the asymmetric sine-Gordon model as well [45, 46], can be also investigated from the point of view of the entropic information and the brane-world configurational entropy.

### Acknowledgments

RACC thanks to UFABC and CAPES for financial support. RdR thanks to SISSA for the hospitality and to CNPq Grants No. 303027/2012-6 and No. 473326/2013-2 for partial financial support. RdR is also supported in part by the CAPES Proc. 10942/13-0.

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